**Barron’s Let’s Review Regents – Algebra II**

# Chapter 2: Rational Expressions and Equations

## 2.1 Arithmetic With Rational Expressions

**Key Ideas**

A *rational expression* is a fraction that has a polynomial expression in the denominator. It often also has a polynomial expression in the numerator.

An example of a rational expression is . Just like fractions involving integers, rational expressions can be simplified, reduced, multiplied, divide, added, and subtracted.

**Reducing Rational Expressions**

A rational number is a fraction, like , that has an integer in both the numerator and the denominator. When both the numerator and denominator are multiplied or divided by the same number, the result is a rational number that is equivalent to the original number.

*Reducing* a rational number is when the numerator and denominator are both divided by the same common factor. Factoring the numerator and denominator of a rational number makes it easier to reduce the fraction to *lowest terms*.

**Multiplying Rational Expressions**

**Dividing Rational Expressions**

**Adding Rational Expressions**

**Situation 1:** The expressions already have a common denominator.

**Situation 2:** One denominator is a multiple of the other denominator.

**Situation 3:** The denominators have no common factor.

If the denominators have no common factor, the lowest common denominator is the product of the two denominators.

Situation 4: The two denominators share a common factor, but the denominator is not a multiple of the smaller one.

**Subtracting Rational Expressions**

Subtracting rational expressions is nearly the same as adding them. An extra complication, which happens frequently on the Regens exam, is you must be careful distributing the negative sign through the parentheses of the second expression.

### Check Your Understanding of Section 2.1

1. Multiple-Choice
2. What is reduced to simplest terms?  
   **(1)**
3. What is reduced to simplest terms?  
   **(3)**
4. What is reduced to simplest terms?  
   **(1)**
5. What is reduced to simplest terms?  
   **(3)**
6. What is reduced to simplest terms?  
   **(4)**
7. When is multiplied, which of the following does it have the same answer as?  
   **(4)**
8. What is **(2)**
9. What is =   
   **(1)**
10. What is   
    **(2)**
11. What is ?  
       
       
       
    **(3)**
12. *Show how you arrived at your answers*.
13. Ethan says that can be reduced to . Braylon says this is not correct. Who is right and why?  
      
    Braylon is correct . The constant 5 was not divided by 2. The correct answer is .
14. James notices the following pattern:  
    He has a theory that, in general,   
    . Prove that James is correct about his theory?
15. Simplify .
16. Talia simplified by this process:  
       
    There was an error in Talia’s calculation. What was the error?  
      
    The should have been . The multiplication by -2 should have been distributed throughout the term.
17. Fully simplify .
18. The rational expression can be reduced. What is it in fully reduced form?

## 2.2 Solving Rational Equations

**Key Ideas**

A rational equation is an equation contains at least one rational expression. Rational equations often require finding a common denominator for all the terms involved. The process for solving a rational equation often produces extra solutions that need to be rejected.

**Simple Rational Equations**

The simplest rational equations are ones that already have a common denominator.

Since these have the same denominator already, the denominators can be ignored to create the equation with just the numerators.

With rational equations, it is necessary to check your answer in case the “ignoring” step somehow caused an incorrect answer to creep in. Simply plug your answer into the original problem.

**Solving Rational Equations with Cross Multiplication**

Some rational equations have just one term on each side but the terms have different denominators.

The quickest way to solve an equation of this form is by cross multiplication. To cross multiply, make a new equation that has the product of one numerator with the other denominator on each side of the equal sign.

To check if this is correct, substitute into the original equation.

**Math Facts**

Cross multiplication is a shortcut for making both sides of the equal sign have a common denominator. You then ignore the denominators and make an equation out of the numerators. If , use cross multiplication to get the denominator bd: or .

**Solving Rational Equations with Multiple Terms**

If the rational equation has more than one term on either side of the equal sign (normally the left side), those terms need to be combined. Then the resulting equation can be solved using the methos mentioned earlier in this section.

The process of ignoring the denominator can result in fake answers. If the denominator would become 0 with a solution, that solution must be rejected (because division by 0 makes it undefined).

**Rate Word Problems**

Some real-world word problems involving rates can be solved using rational equations.

### Check Your Understanding of Section 2.2

1. Multiple-Choice
2. Solve for .  
   **(3) 4**
3. Solve for .  
   **(4) 5**
4. Solve for .  
   **(3) 2**
5. Solve for .  
   **(3) or**
6. Solve for .  
      
      
      
      
      
   **(3) 4**
7. Solve for .  
      
      
   **(2) 5**
8. Solve for .  
      
      
      
   **(1) or**
9. Solve for .  
      
      
      
      
   **(1) -10**
10. Solve for .  
       
       
       
       
    **(4) 8**
11. Solve for .  
       
       
       
       
       
       
       
       
    **(1) or**
12. *Show how you arrived at your answers*.
13. A group of people contribute equal amounts of money to get a $24 gift for a friend. If two or more people contributed and they all paid equal amounts, they would pay $1 less. How many people were there originally?  
    x = number of people originally?  
    x + 2  
     **(number of original people, cost $4 each)  
    x, cost per person is $3 each.**
14. Leah solves   
     and gets solutions of and . Paxton says that just is the solution.  
      
    **Paxton is correct, because the original equation has an undefined value for .**
15. Kevin and Noelle drive 240 miles to Boston to watch a Yankees and Red Sox game. ON the way home, they drive 10 mph slower and get home 48 minutes later than they would have had they driven the same speed as before. How fast did they drive from New York to Boston?  
    x is the original rate of speed in miles per hour. 240/x is the amount of time in hours it took to travel 240 miles.  
    Multiply by 1.25
16. What is the solution to:  
     ?  
      
       
       
    Divide by 4  
     **or**
17. Solve for .

## 2.3 Graphing Rational Functions

**Key Ideas**

Just like polynomial, exponential and radical functions, rational functions can be graphed. The graphs of rational functions usually include horizontal and/or vertical asymptotes. Asymptotes are invisible lines that curves get closer and closer to. The graph of a rational function can help solve certain rational equations.

**Vertical and Horizontal Asymptotes**

Below is the graph of the rational function  
 .

A graph of a function

AI-generated content may be incorrect.

A characteristic of a graph of a rational function is that it often has one or more *vertical asymptotes*. A vertical asymptote is a vertical line that the graph never touches or crosses. In this graph, the vertical asymptote is the vertical line at .

Some graphs of rational functions also have horizontal asymptotes Like the vertical asymptote, it is a horizontal line that, usually for very high and very lose -values, the curve gets closer and closer to without touching or crossing.

**Checking to See if Two Rational Expressions Are Equivalent Using the Graphing Calculator**

**Solving Rational Equations with the Graphing Calculator**

The equation , which was solved with algebra earlier in this chapter, can be solved with the intersect feature of the graphing calculator.

A math problem with numbers

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A graphing of a function

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The solution is

### Check Your Understanding of Section 2.3

1. Multiple-Choice
2. What are the asymptotes of the graph of   
   ?  
   **(2)**
3. Which of the following could be the equation for this graph?  
   Asymptotes:   
   horizontal: y = 0, for large values of x  
   vertical: x = -2  
   **(1)**
4. Which of the following is the graph of ?  
   Asymptotes: y = 0, for large values of x  
   x = -2  
   y-intercept: (0, 4)  
   **(3)**
5. Use a graphing calculator to determine which equation has the same graph as .  
   **(1)**
6. Which of the following has the same graph as   
   **(2)**